Vector Calculus
Quadric Surface Classification

Dr. Paul L. Bailey

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A *quadric surface* is the locus of an equation of the form

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + Gxy + Hxz + Iyz = J.$$

We may eliminate the mixed terms by rotations; thus assume that the equation is of the form

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz = J.$$

In the presence of a quadratic (degree 2) term we may eliminate the linear (degree 1) term via completing the square to obtain a translation; thus assume that each variable occurs as a quadratic or as a linear term, but not both.

We call J the *constant*, and assume that $J \geq 0$.

A *slice* is the intersection of a quadric surface with a plane perpendicular to a coordinate plane.

We classify the surfaces as follows.

- If all variables are linear, it is a *plane*.
- If one variable is missing, it is a *cylinder*.
- If two variables are linear, it is a *cylinder*.
- If two variables are quadratic and one is linear, it is a *paraboloid*. In this case, two slices are parabolas.
  - If the quadratics have the same sign, it is an *elliptic paraboloid*.
  - If the quadratics have different signs, it is a *hyperbolic paraboloid*, or "saddle".
- If three variables are quadratic and positive, it is an *ellipsoid*.
- If three variables are quadratic and one has a different sign, it is a *hyperboloid*. In this case, two slices are hyperbolas.
  - If two quadratics are positive, it is a *one-sheeted hyperboloid*.
  - If two quadratics are negative, it is a *two-sheeted hyperboloid*.

We give an example equation for each type.

- Plane $x + y + z = 1$
- Cylinder $z = x^2 + y$
- Elliptic Paraboloid $z = x^2 + y^2$
- Hyperbolic Paraboloid $z = x^2 - y^2$
- Ellipsoid $x^2 + y^2 + z^2 = 1$
- One-Sheeted Hyperboloid $x^2 + y^2 - z^2 = 1$
- Two-Sheeted Hyperboloid $z^2 - x^2 - y^2 = 1$